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LETTER TO THE EDITOR

TURÁN'S THEOREM AND MAXIMAL DEGREES

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The number of the edges of a graph G will be denoted by $e(G)$, and the subgraph induced by the neighbours of the vertex x – by G_x. For the n-vertex r-partite Turán's graph $T_r(n)$ we define $e\big(T_r(n)\big) = t_r(n)$.

B. Bollobàs in [1] and [2] considered the following simple algorithm to construct a large clique in a graph G: "Pick a vertex x_1 of maximal degree in $G_1 = G$, then a vertex x_2 of maximal degree in $G_2 = G_{x_1}$, and so on. The algorithm stops with x_l if x_l has no neighbours in G_l". In [1] this algorithm was called the degree-greedy algorithm.

B. Bollobas in [1] proved the following results:

Theorem 2 (see also Theorem 5 in [2]). *Let G be a graph with n vertices and $t_r(n) + a$ edges, where $a \geqq 0$. Let x be a vertex of maximal degree d. Then $e(G_x) \geqq t_{r-1}(d) + a$, and the inequality is strict unless $n - d = \left\lfloor \frac{n}{r} \right\rfloor$, and $G = G_x + \overline{K}_{n-d}$.*

Theorem 5 (see also Theorem 6 in [2]). *Let G be a graph with n vertices and least $t_r(n)$ edges. Then either $G = T_r(n)$ or else the degree-greedy algorithm constructs a clique of G of order at last $r + 1$.*

The aim of this note is to draw attention to the fact that:

1. Theorem 2 is a special case of Proposition 1, p. 235 in [3] ($\langle A \rangle = G_x$ for some vertex x of maximal degree), because the case $a = 0$ of Theorem 2 is obviously equivalent to Theorem 2.

2. The degree-greedy algorithm found by B. Bollobas, A. Thomason and J. Bondy in 1983 is a fact published in [4], p. 119. by N. Khadzhiivanov and N. Nenov in 1976.

3. Theorem 5 coincides with Corollary 1 from [4], p. 121.

4. Theorem 3 in [1] is not correct, because $\triangle\big(T_r(n)\big) \neq (r-k-1)s+p$.

5. In the proof of Theorem 5, given in [1] and [2], equalities $G_x = T_{r-2}(n-k)$ and $G = T_{r-1}(n)$ are not correct.

REFERENCES

1. Bollobás, B. Turán's Theorem and Maximal Degrees. *J. Combin. Theory Ser. B*, **75**, 1999, 160-164.
2. Bollobás, B. Modern Graph Theory. Springer-Verlag, New York, 1998.
3. Hadjiivanov (Khadzhiivanov), N., N. Nenov. On the Generalized Thurán's Theorem and extension of this Theorem. *Ann. Sof. Univ., Fac. Math.*, **77**, 1983, 231-242 (in Russian).
4. Hadjiivanov (Khadzhiivanov), N., N. Nenov. Extremal problems for s-graphs and the theorem of Turan. *Serdica*, **3**, 1977, 117-125 (in Russian).

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