

## FREQUENCY MODULATED SIGNAL STANDING OUT BY STOCHASTIC RESONANCE EFFECT

OKSANA KHARCHENKO AND ZLATINKA KOVACHEVA

In this paper, the phenomenon which is known as stochastic resonance is considered to the signal processing application. Signal standing out with the presence of noise is considered to be one of the basic problems of telecommunication and radioengineering. The stochastic resonance effect is shown to provide significant improvement of some characteristics of the information signal, such as power gain, and noise dispersion at the system output at a certain optimal noise level. In the present article, Minimum Shift Keying signal mixed with Gaussian white noise has been studied using stochastic resonance effect. Noise coefficient, which is one of the quantitative characteristics of noise immunity is calculated and investigated.

**Keywords:** stochastic resonance, Minimum Shift Keying, signal standing out, noise coefficient, noise dispersion, data engineering

**CCS Concepts:**

- Applied computing~Physical sciences and engineering

### 1. INTRODUCTION

The main and the most difficult problem of the signal receiving is the problem of noise immunity. The problem of finding the best methods for radio signal receiving with the noise presence is a one of the main goals in radio engineering and telecommunication [12–14].

The researches made in 1980s brought us to paradoxical conclusions. Noise can help amplify the signal. This phenomenon is known as stochastic resonance (SR) [4, 7, 10, 16]. The SR effect presents the response to the weak input signal, in a non-linear system. The system parameters, such as power gain and signal-to-noise ratio (SNR), have their maximum in specific conditions. Nowadays, this effect is

fundamental and has been observed in biology, medicine, psychology, neuroscience, etc. [2, 6, 8, 18].

SR is described by the following equation (see [1]):

$$dy/dt = y(t) - y^3(t) + x(t) + n(t),$$

where  $x(t) + n(t)$  is an input process being additive mixture of the useful signal and the white normal noise, and  $y(t)$  is the output signal of the non-linear system.

This equation is an Abel equation of first order and it does not have an analytical solution [9].

## 2. SHIFT KEYING SIGNAL STANDING OUT FROM THE ADDITIVE MIXTURE WITH NOISE

Currently, telecommunication systems process digitally modulated radio signals. We can cite, as an example, the Decree of the Government of Bulgaria on the widespread use of digital television [11].

Digital modulation is the process of converting digital symbols into signals that are compatible with the characteristics of the channel. Pulses of a given shape modulate the sine wave. Types of modulation depend on the parameter of the radio signal, which varies in proportion to the envelope. Since the digital signal is a sequence of rectangular pulses, and the parameter of the carrier oscillation at a certain point in time changes by a jump, therefore, modulation is actually a manipulation [17].

Currently, the GSM standard uses Minimum Shift Keying (MSK). The primary objective of spectrally efficient modulation techniques is to maximize bandwidth efficiency. Therefore, MSK is widely used in digital telecommunication systems [5, 15].

The modulation of the MSK is as follows: The original data stream consists of bipolar pulses. This pulse stream is divided into an in-phase stream  $d_L(t)$  (even bits) and a quadrature stream  $d_Q(t)$  (odd bits). The MSK waveform can be expressed as [15]:

$$s(t) = d_L(t) \cos \frac{\pi t}{2T} \cos 2\pi f_0 t + d_Q(t) \sin \frac{\pi t}{2T} \sin 2\pi f_0 t, \quad (2.1)$$

where  $T$  is the width pulse and  $f_0$  is the carried frequency.

Let us determine the MSK signal for the pulsed data stream:  $-1; 1; 1; -1; -1; 1; -1; 1$ . Figure 1 presents equation (2.1) for this signal. Figure 1a and c illustrate the sinusoidal weighting of the  $L$ - and  $Q$ -channel pulses, respectively. Figure 1b and d show the modulation of the orthogonal components  $\cos \omega_0 t$  and  $\sin \omega_0 t$ , respectively. Figure 1e illustrates the summation of the orthogonal components from Figure 1b and d.

By adding white Gaussian noise to this signal, we apply the stochastic resonance effect to stand out the useful signal. The result of this signal processing is shown in Figure 2.

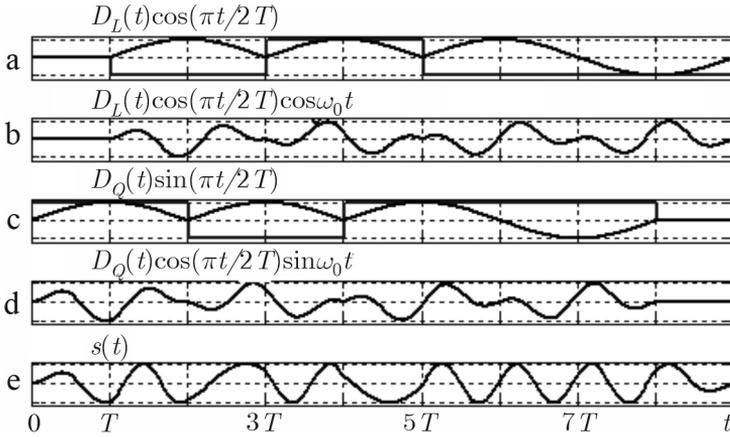


Figure 1. MSK waveform

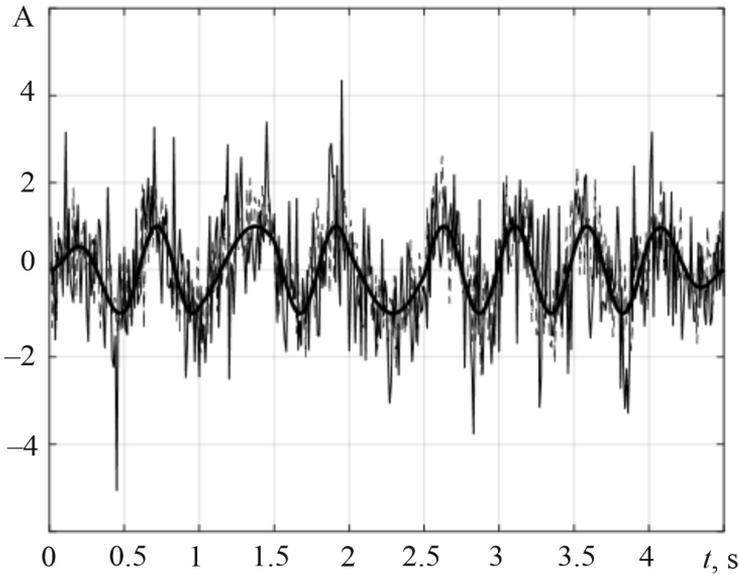


Figure 2. Standing out of the signal from additive signal-noise mixture

It can be seen from Figure 2, that it is possible to significantly reduce the noise component of the oscillations by SR effect. In this case, the input noise dispersion is 1 p.u., the output noise dispersion is 0.4 p.u. The signal is amplified by the SR effect and the power gain is  $k = 1.3$ .

In Figure 2, input MSK signal is shown by the thick line, signal-noise mixture is illustrated by the thin line, and output signal by the dotted line.

Thus, the SR effect provides significant noise suppression and amplification of the useful signal in communication systems with MSK modulation. Figure 3, which

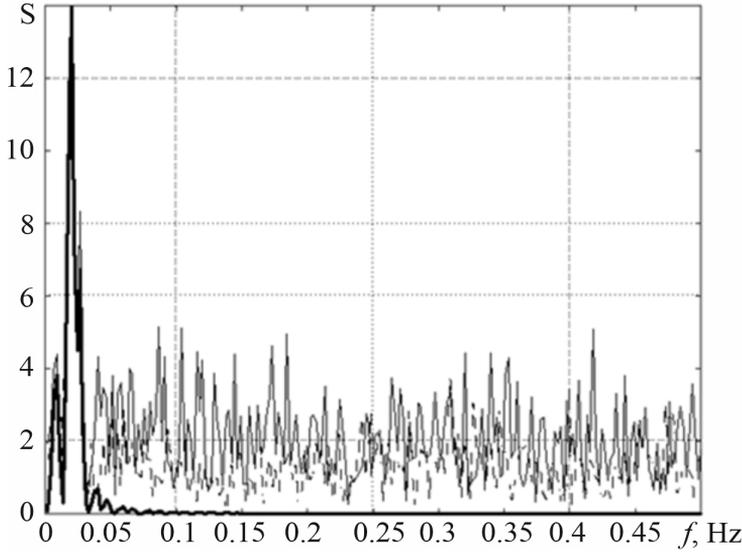


Figure 3. Amplitude spectra (input MSK signal – thick line, signal-noise mixture – thin line, output signal – dotted line)

shows the spectra of these signals, also confirms the effective stand out of the MSK signal.

One of the quantitative characteristics of noise immunity is the noise coefficient. It is defined as a ratio of input signal-to-noise ratio ( $SNR_{input}$ ) to output signal-to-noise ratio ( $SNR_{output}$ ) [3].

$$L = SNR_{input}/SNR_{output}. \tag{2.2}$$

We can rewrite this formula in the following form:

$$L = \frac{SNR_{input}}{SNR_{output}} = \frac{1}{K_p K_N},$$

where:

$K_p = \frac{P_{s_{output}}}{P_{s_{input}}}$  is the power gain;

$P_{s_{input}}$  is the input power;

$P_{s_{output}}$  is the output power;

$K_N = \frac{D_{input}}{D_{output}}$  is the noise reduction coefficient;

$D_{input}$  is the input noise dispersion;

$D_{output}$  is the output noise dispersion.

From the result, it is seen, that the decrease of the noise coefficient occurs either as a result of an increase in power gain due to an increase in the power of the output signal, or as a result of an increase in the noise reduction coefficient due to a

decrease in the output noise level, or as a result of both factors simultaneously. In fact, all these cases present the SR effect, which consists in the transition of noise energy into the energy of a useful signal.

In Figure 2 and 3,  $L \approx 0.31$ , and the signal is amplified. Therefore, the power gain is greater than 1, the noise reduction factor is also greater than 1 due to the decrease of the output noise level.

### 3. CONCLUSION

Stochastic resonance is just beginning to be applied in telecommunication and radioengineering. In this work, the SR effect is used for the first time for standing out of digital signals of the telecommunication systems against the background noise.

The SR effect is investigated, which makes it possible to stand out a weak (compared to noise) signal from an additive mixture of signal and white Gaussian noise.

We showed that SR effect stands out the MSK signal, and provides a significant reduction of the noise component of the signal. As a result, there is an input noise dispersion equals 1 p.u., and the output noise dispersion equals 0.4 p.u. Simultaneously, the signal is amplified by the SR effect and the power gain  $k = 1.3$ , noise coefficient  $L \approx 0.31$ .

It is noted, that the decrease of the noise coefficient occurs either as a result of the increase of the power gain, due to the increase of the output signal power, or as a result of the increase of the noise reduction coefficient, due to the decrease of the output noise level.

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